APPENDIX C

A Formal Spatial Price Equilibrium Model

This appendix presents the formal conceptual spatial price equilibrium model employed in the UMR-IW Navigation System Feasibility Study. The model consists of a set of relationships and postulates regarding economic agents' behaviors.

Notation

SETS

J = The set of regions.

K = The set of intermediate goods.

N = The set of final consumption goods.

$L_{i,j}$ = The set of transportation paths from region i to region j.

QUANTITIES

$PF_{n,i}$ = The quantity produced of final good n in region i.

$CF_{n,i}$ = The quantity consumed of final good n in region i.

$PI_{k,i}$ = The quantity produced of intermediate good k in region i.

$XF_{n,i,j,l}$ = The flow of final good n from region i to region j on transportation path l.

$XI_{k,i,j,l}$ = The flow of intermediate good k from region i to region j on transportation path l.

FUNCTIONS

$P^s_{k,i}$ = The inverse supply function of intermediate commodity k in region i.

$P^d_{n,i}$ = The inverse demand function for final commodity n in region i.

$t^s_{k,i,j,l}$ = The inverse supply function of transportation of intermediate commodity k from region i to region j on transportation path l.

$t^s_{n,i,j,l}$ = The inverse supply function of transportation of final commodity n from region i to region j on transportation path l.

$c_{n,i}$ = The cost of transformation function for production of final good n in region i.

The production function of each region is given by the implicit transformation function:

$$f_i\left(\sum_j \sum_l XI_{k,j,i,l}, \forall k; PF_{n,i}, \forall n\right)$$

Using the primal approach, it may be demonstrated that a market equilibrium is achieved through the maximization of a complex function constructed from the transportation and regional supply functions and regional demand functions. This function is often referred to as the net social payoff function because under certain market conditions the constructed function itself yields the sum of producer surplus and consumer surplus. In a multi-regional, multi-commodity context, the net social payoff function is obtained by summing the net social payoff functions across regions and commodities.

The maximization of the net social payoff function is performed subject to two sets of constraints termed the trade flow constraints and the non-negativity constraints. The trade flow constraints formalize the idea that each region cannot use domestically and export more than the amount it produces, and each region cannot consume more of a good than it produces and imports. The non-negativity constraints formalize the notion that no region can produce, consume, or trade negative quantities of any good.

The Formal Maximization Problem

The identification of a spatial equilibrium for this model can be achieved by solving the following constrained maximization problem.

Maximize:

$$\sum_i \left\langle \sum_n \int_0^{CF_{n,i}} p^d_{n,i}(t)dt - \sum_k \int_0^{PI_{k,i}} p^s_{k,i}(u)du - \sum_n \int_0^{PF_{n,i}} c_{n,i}(v)dv \right\rangle$$

$$-\sum_k \sum_i \sum_j \sum_l \int_0^{XI_{k,i,j,l}} t^s_{k,i,j,l}(w)dw - \sum_n \sum_i \sum_j \sum_{lj} \int_0^{XF_{n,i,j,l}} t^s_{n,i,j,l}(z)dz$$

subject to:

$$PI_{k,i} \geq \sum_j \sum_l XI_{k,i,j,l}, \forall k,i;$$

$$f_i\left(\sum_j \sum_l XI_{k,j,i,l}, \forall k; PF_{n,i}, \forall n\right) \leq 0, \forall i;$$

$$PF_{n,i} \geq \sum_j \sum_l XF_{n,i,j}, \forall i,n;$$

$$\sum_j \sum_l XF_{n,j,i,l} \geq CF_{n,i}, \forall i,n;$$

$$PI_{k,i} \geq 0, \forall k,i;$$

$$PF_{n,i} \geq 0, \forall n,i;$$

$$CF_{n,i} \geq 0, \forall n,i;$$

$$XP_{k,i,j,l} \geq 0, \forall k,i,j,l; and$$

$$XF_{n,i,j,l} \geq 0, \forall n,i,j,l.$$

Identification of Equilibrium Conditions

The equilibrium is identified applying the Kuhn-Tucker theorem for constrained maximization. First, the Lagrangian function is formed. Then, the first order conditions for maximization of the objective function are derived, and, finally, the equilibrium conditions are interpreted.

Application to UMR-IW Navigation System Study

The general results described above and some further simplifying assumptions yield the model as implemented in the Excel Spreadsheet. Appendix A presents details on the IN ESSENCE spreadsheet model.